Acceleration of particles and shells by Reissner-Nordström naked singularities

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Abstract

We explore here the Reissner-Nordström naked singularities from the perspective of the particle acceleration. We first consider a collision between the test particles following the radial geodesics in the Reissner-Nordström naked singular geometry with $Q > M$. An initially radially ingoing particle turns back due to the repulsive effect of gravity in the vicinity of naked singularity. Such a particle then collides with an another radially ingoing particle. We show that the center of mass energy of collision taking place at $r \approx M$ is unbound, in the limit where the charge transcends the mass by arbitrarily small amount $0 < 1 - M/Q \ll 1$. The acceleration process we described avoids fine tuning of the parameters of the particle geodesics for the unbound center of mass energy of collision and the proper time required for the process is also finite. We then study the collision of the neutral spherically symmetric shells made up of dust particles. In this case, it is possible to treat the situation exactly taking into account the gravity due to the shells using Israel's thin shell formalism, thus allowing us to go beyond the test particle approximation. The center of mass energy of collision of the shells is then calculated in a situation analogous to the test particle case and is shown to be bounded above. However, we find that the center of mass energy of a collision between two of constituent particles of the shells can exceed the Planck energy.

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I. INTRODUCTION

Since the terrestrial particles accelerators like Large Hadron Collider probe particle physics at the energy scales that are almost 16 orders of magnitude smaller than the Planck scale, it would interesting to investigate whether or not various naturally occurring high energy astrophysical phenomenon could shed light on the new physics at higher energy scales that remain unexplored. Stepping ahead towards this exiting possibility an interesting proposal was made recently which suggests that the Kerr black holes could act as particle accelerators[1]. It was shown that the two particles dropped in from infinity at rest, traveling along the timelike geodesics can collide and interact near the event horizon of a Kerr black hole with divergent center of mass energy, provided the black hole is close to being extremal and angular momentum of one of the particles takes a specific value of the orbital angular momentum. The possible astrophysical implications of this process in the context of annihilations of the dark matter particles accreted from the galactic halo, around the event horizon of the central supermassive black hole, were also investigated [2]. This process of particle acceleration suffers from several drawbacks and limitations pointed out in[3]. The angular momentum of one of the colliding particle must take a single fine tuned value. The proper time required for the particle with fine tuned angular momentum to reach the horizon and thus the time required for the collision to take place is infinite. The backreaction and the gravitational radiation emitted by the colliding particles was neglected. There were many investigations of this acceleration mechanism in the background of Kerr as well as many other black holes[4].

We studied and extended the particle acceleration mechanism to the Kerr naked singular geometries transcending Kerr bound by arbitrarily small amount $0 < a - 1 \ll 1$ [5]. We considered two different scenarios where the colliding particles follow a geodesic motion along the equatorial plane as well as along the axis of symmetry of the Kerr geometry. In the first case, the particles are released from infinity at rest in the equatorial plane. One of the initially infalling particle turns back as an outgoing particle due to its angular momentum. It then collides with an another infalling particle around $r = 1$. We showed that the center of mass energy of collision between these two particles is arbitrarily large. The angular momentum of the colliding particles is required to be in a finite range as opposed to the single fine tuned value in case of Kerr black holes. Thus the extreme fine tuning of the angular momentum is avoided in such a collision. The proper time required for such a collision to take place is also shown to be finite. In the second case, the particles are released from rest along the axis of symmetry, from large but finite distance. These particles have zero angular momentum. One of the particles initially falls in and then turns back due to the repulsive effect of gravity in the vicinity of a Kerr naked singularity. This particle then collides with an ingoing particle at $z = 1$. The center of mass energy of collision is arbitrarily large and the proper time required for the process to take place is finite. Thus two issues related to acceleration mechanism in Kerr black hole case, namely

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the fine tuning of the angular momentum and the infinite time required for the collision, are avoided in case of Kerr naked singularities.

The issue of the backreaction and gravitational radiation due to the point particle is difficult to deal in general. The accretion of the particles onto an astrophysical object can be expected to be more or less isotropic in many cases. Thus it would be interesting and more physical to study the motion and collisions of the shells of particles instead. The rigorous mathematical analysis of the shells would be very extremely difficult in the Kerr spacetime due to the lack of sufficient symmetry. The motion and collision of the spherical shells would be exactly tractable in the spherically symmetric spacetimes following the Israel‘s thin shell formalism[6]. We first note that while no gravitational radiation is emitted by a perfectly spherical shell, the gravitational radiation per particle emitted by a quasispherical shell of particles will be significantly lower than the radiation emitted by a single particle. Thus it might be reasonable to ignore the gravitational radiation effects and focus entirely on the backreaction while dealing with the shells.

The acceleration of the particles around the extremal Reissner-Nordström black hole was studied in [8],[9]. This process is mathematically similar to the acceleration process in Kerr geometry. The center of mass energy of collision near the horizon of the extremal Reissner-Nordström black hole, of the charged and uncharged particles is shown to be divergent. The collision of the charged and uncharged spherical shells was investigated in[9]. The dynamics of the shells when their gravity is ignored is same as that of the test particles. Whereas when the exact calculation is done taking into account the backreaction effects, the center of mass energy turns out to be finite. Thus it was speculated that the center of mass energy of collision of particles around Kerr black hole might also turn out to be finite when the gravity due to the colliding particles is taken into account.

In this paper we first describe the particle acceleration process in the background of Reissner-Nordström naked singularities. We show that the center of mass energy of collision between two uncharged particles, one of then initially ingoing and other one initially ingoing, but turning back due to the repulsive effect of gravity in the vicinity of naked singularity is arbitrarily large, when the collision happens around $r \approx M$, provided that the deviation of the Reissner-Nordström charge from the mass is extremely small. We then investigate the collision between two uncharged shells made up of dust particles, in a situation analogous to the particle collision, taking into account their gravity. We find that the center of mass energy of a collision between the shells is bounded above. However, the center of mass energy of a collision between two of constituent particles of the shells can exceed the Planck energy which might be a threshold value of the quantum gravity.

II. ACCELERATION OF PARTICLES BY REISSNER-NORDSTRÖM NAKED SINGULARITIES

The line element of the Reissner-Nordström geometry is the spherical coordinates (t, r, θ, ϕ)given by

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega^2 \qquad (1)$$

where $d\Omega^2$ is a metric on the unit two-sphere. It is a unique solution of Einstein equations under the assumptions of spherical symmetry, asymptotic flatness with the electromagnetic field as a source of spacetime curvature. The metric function $f(r)$ is given by

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \qquad (2)$$

This solution contains two parameters M and Q, namely the mass and charge associated with Reissner-Nordström geometry. In this paper, we assume that M and Q are positive.

The event horizon in the Reissner-Nordström geometry if it exists is given by a solution to the equation $f(r) = 0$. There are two roots to this quadratic equation given by $r = M \pm \sqrt{M^2 - Q^2}$. There are two real roots to the equation when $M^2 > Q^2$. The larger root is the event horizon and the geometry correspond to the Reissner-Nordström black hole. When $M^2 = Q^2$ both the roots of coincide. In this case the black hole is known as the extremal black hole with event horizon at $r = M = Q$. Whereas in the case when $Q^2 > M^2$ there is no real root to the equation $f(r) = 0$. Thus the event horizon is absent and the timelike singularity at $r = 0$ is exposed to the asymptotic observer at infinity. This configuration thus contains a globally visible naked singularity. We will investigate the last case in this paper from the perspective of particle acceleration.

Before proceeding further it is worthwhile to mention that, the naked singularities are associated with pathological features like the breakdown of predictability and and so on. That was precisely the reason Penrose came up with the cosmic censorship conjecture abandoning the existence of naked singular solutions in our universe[10]. However there were recent developments in the framework in string theory, which suggests by means of the specific worked out examples, that the naked singularities might be resolved by high energy stringy modification to the classical general relativity [11] and various pathological features disappear. This renders the classical naked singular solutions legal as long as one stays sufficiently away from high curvature region where quantum gravity would prevail.

We now study the motion of a point test particle following a timelike geodesic in the Reissner-Nordström naked singular geometry with $Q^2 > M^2$. Let U^μ be the velocity of the particle. The motion of the particle is confined to the plane owing to the spherical symmetry which can be

chosen to be equatorial plane $\theta = \frac{\pi}{2}$ using the gauge freedom. Various metric components (1) are manifestly independent of time coordinate and azimuthal angular coordinate coordinate, depicting the existence of the Killing vectors ∂_t and ∂_ϕ. The following quantities are conserved along the geodesic of the particle $E = -U \cdot \partial_t$

and $L = U \cdot \partial_\phi$,

where E can be interested as the conserved energy of the particle per unit mass and L can be interpreted as the conserved angular momentum of the particle per unit mass. Using these constants of motion and the normalization condition for velocity of the particle the components of the velocity of the particle can be written as

$$\begin{aligned} U^t &= \frac{E}{f} \\ U^r &= \pm\sqrt{E^2 - f\left(1 + \frac{L^2}{r^2}\right)} \\ U^\theta &= 0 \\ U^\phi &= \frac{L}{r^2} \end{aligned} \tag{3}$$

$\pm$ stands for the radially outgoing and infalling particles respectively. The equation yielding the radial component of velocity can also be written in the following form

$$U^{r2} + V_{\text{eff}}(r, L) = E^2 \tag{4}$$

The quantity V_{eff} can be thought of as a effective potential for motion in the radial direction in analogy with the celebrated energy conservation equation in the classical mechanics. The effective potential is given by the expression

$$V_{\text{eff}} = f\left(1 + \frac{L^2}{r^2}\right) \tag{5}$$

For simplicity and from the perspective of the comparison to shell collision that would be discussed in the next section, we make a simplifying assumption that the angular momentum of the particle is zero $L = 0$. This implies that the motion of the particle us purely radial. The effective potential now can be written as

$$V_{\text{eff}} = f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \tag{6}$$

The effective potential as a function of radius r is plotted in Fig.1. For large values of radial coordinate as $r \to \infty$, $V_{\text{eff}} \to 1$. Whereas as one approaches the naked singularity $r \to 0$, effective potential blows up $V_{\text{eff}} \to \infty$.It always remains greater than zero.It admits a minimum for an intermediate value of r which is given by

$$\begin{aligned} r_{\text{min}} &= \frac{Q^2}{M} \\ V_{\text{eff,min}} &= 1 - \frac{M^2}{Q^2} \end{aligned} \tag{7}$$

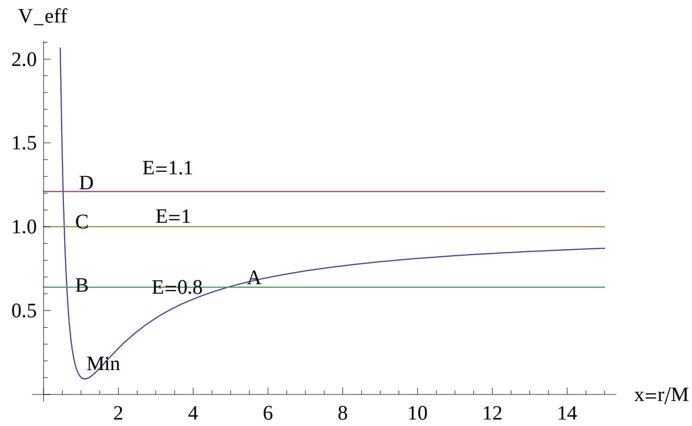

FIG. 1: The effective potential is plotted against $x = \frac{r}{M}$for a particle following radial geodesic in Reissner-Nordström naked singular geometry with $\frac{Q}{M} = 1.05$. It admits a minimum at the classical radius $x = \frac{Q^2}{M^2}$, depicted by 'min', where gravity changes it's character from being attractive to repulsive in the close neighborhood of singularity. The ingoing particle thus gets reflected back as an outgoing particle close to singularity. The particle having energy $E = 0.8 < 1$ is bound and oscillates between points A and B. Particle with energy $E = 1.1 > 1$ is unbound, has only one turning point D. Particle with $E = 1$ is marginally bound, has a turning point at C. The potential energy curve asymptotes to the $E = 1$ as $x \to \infty$.

Note that r_{min} coincides with the classical radius associated with an object of charge Q and mass M. It is clear from the shape and slope of the effective potential curve that the gravity of the Reissner-Nordström naked singularity is attractive in the regime $r \in \left(r_{\text{min}} = \frac{Q^2}{M}, \infty\right)$, from the classical radius all the way upto infinity. Whereas the gravity is repulsive in the region extending from the singularity to the classical radius $r \in \left(0, r_{\text{min}} = \frac{Q^2}{M}\right)$. The nature of gravity changes from attractive to repulsive in the vicinity of the naked singularity. Similar behavior is also observed in case of other known examples of the stationary naked singularities[12]. An ingoing particle at initially speeds up upto the classical radius. It then slows down due to the repulsive gravity and gets reflected back eventually. It them emerges as an outgoing particle.

If the conserved energy of the particle is less than unity $E < 1$ then the particle is bound, i.e. it oscillates back and forth between the radii $r = \frac{M\left(1 - \sqrt{1 - \frac{Q^2}{M^2}(1 - E^2)}\right)}{1 - E^2}$ and $r = \frac{M\left(1 + \sqrt{1 - \frac{Q^2}{M^2}(1 - E^2)}\right)}{1 - E^2}$, which are turning points for the particle where the radial velocity is zero $U^r = 0$. If the conserved energy is identical to unity $E = 1$, then there is only one turning point given by $r = \frac{Q^2}{2M}$. The particle is at rest at infinity as the radial velocity approaches zero asymptotically. The particle is said to be

marginally bound. Whereas in the case when energy is larger than unity $E > 1$, again there is only one turning point given by $r = \frac{M\left(-1+\sqrt{1-\frac{Q^2}{M^2}(1-E^2)}\right)}{E^2-1}$. The asymptotic velocity of the particle as it reaches infinity is positive $U^r \to \sqrt{E^2-1}$. Such a particle trajectory is unbound. All three cases are shown in Fig. 1.

Yet another interesting feature that is evident from the Fig. 1 is that the effective potential admits a minimum at the classical radius $r = \frac{Q^2}{M}$. This implies that the particle can stay at rest at this radius in stable equilibrium. Such a freely floating particle has a conserved energy $E = \sqrt{1-\frac{M^2}{Q^2}}$.

We now consider a collision between two particles moving along the radial geodesics, each with mass m and conserved energy $E = 1$ and zero angular momentum L. Particles are assumed to be marginally bound. They are released from rest from infinity. One could replace marginally bound particles by either unbound or bound particles. It does not change the conclusions. Let U^1 and U^2 be their velocities. We assume that one of the particles is initially ingoing particle which eventually shows down and then turns back as an outgoing particles due to the repulsive effect of gravity in the vicinity of the singularity. Such a particle then collides with another ingoing particle at the radial coordinate r. The velocities of the two colliding particles are given by

$$U^1 = \left(\frac{1}{f}, \sqrt{1-f}, 0, 0\right)$$
$$U^2 = \left(\frac{1}{f}, -\sqrt{1-f}, 0, 0\right) \tag{8}$$

The center of energy of collision between two particles is given by [1]

$$E_{\text{cm}}^2 = 2m^2(1 - U^1 \cdot U^2) \tag{9}$$

and in this it turns out to be

$$E_{\text{cm}}^2 = \frac{4m^2}{f} \tag{10}$$

The center of mass energy of collision is dependent on the chosen location for the collision, for given values of charge Q and mass M, as the expression above is manifestly dependent on the radial coordinate r through the metric function $f(r)$ which also same as the effective potential for the radial motion V_{eff}. The center of mass energy will be maximum when the effective potential is minimum. The minimum of the effective potential is at the classical radius $r_{\text{min}} = \frac{Q^2}{M}$. If the collision takes place at this location the center of mass energy is maximum and is given by

$$E_{\text{cm,max}}^2 = \frac{4m^2}{1-\frac{M^2}{Q^2}} \tag{11}$$

The maximum value of center of mass energy depends on the ratio of mass to the charge of Reissner-Nordström spacetime. The center of mass energy will be large if the charge transcends the mass by infinitesimally small amount. Here, we introduce a parameter defined by

$$\epsilon := 1 - \frac{M}{Q}. \tag{12}$$

In the limit $\epsilon \to 0$, the center of mass energy of collision between the particles becomes infinite,

$$\lim_{\epsilon \to 0} E_{\text{cm,max}}^2 = \frac{2m^2}{\epsilon} \to \infty. \tag{13}$$

The above equation implies that E_{cm} would be arbitrarily large.

We now briefly mention the subtle differences between the particle acceleration by black holes and naked singularities. In case of the black hole, the divergence of center of energy in the collision has been demonstrated in near extremal or extremal geometries when the mass transcends the charge by arbitrarily small amount $\epsilon \to 0^-$. In this paper, we have shown the divergence of center of mass energy in the naked singular geometry, which can be thought to be near extremal, with the charge transcending the mass by arbitrarily small amount $\epsilon \to 0^+$. In case of black holes, the collision takes place close to the event horizon. It turns out that $V_{\text{eff}} = V_{\text{eff}}' = 0$, as a consequence of which the proper time required for one the particle to reach the horizon and participate in the collision turns out to be infinite. Whereas in case of naked singular geometries, none of these conditions hold good at the point of collision. Thus the proper time required for the collision to take place is finite.

III. ACCELERATION OF SHELLS BY REISSNER-NORDSTRÖM NAKED SINGULAR GEOMETRY

In this section we study the collision of spherical shells in the Reissner-Nordström naked singular geometry. The dynamics of the spherical thin shells becomes tractable exactly owing to the spherical symmetry of the background spacetime. Due to the gravity generated by the shells, the equations describing the motion of shells are no longer that of the test particle following a geodesic in the background Reissner-Nordström geometry.

We deal with the situation that is analogous to the scenario described in the previous section to draw a parallel and compare. We assume that the deviation of the charge from the mass associated with the naked singularity is vanishingly small $0 < \epsilon \ll 1$.

We first describe the procedure to deal with the thin shells taking into account their gravity [6]. We follow notation and convention of Ref. [7]. A shell that is being considered here is the three dimensional submanifold of the four dimensional ambient spacetime manifold with the thin surface layer of matter, i.e., the timelike singular hypersurface . The geometry of the shell can be

described by specifying a three dimensional metric h_{ab} defined over it (also known as the induced metric) and extrinsic curvature, which is a three dimensional tensor K_{ab}, describing how the hypersurface is embedded in the ambient spacetime. The induced metric is continuous while the extrinsic curvature discontinuous across the shell. Discontinuity can be described in terms of the energy-momentum tensor of the shell.

Let Σ be a timelike hypersurface separating spacetime in two regions V_1 and V_2. The coordinates system defined in these two regions be denoted as x_i^μ, $\mu = 0-3$, $i = 1, 2$, whereas the coordinate system defined over the hypersurface be y^a, $a = 0-2$. The projection operator over the three dimensional hypersurface from the four dimensional ambient spacetime is given by

$$e_a^\mu = \frac{\partial x^\mu}{\partial y^a} \tag{14}$$

If $g_{\mu\nu}^i$ is the metric in the ambient spacetime, the induced metric on the hypersurface is given by

$$h_{ab}^i = g_{\mu\nu}^i e_a^\mu e_b^\nu \tag{15}$$

The index i indicates that the calculation of a given quantity is done while approaching the hypersurface from spacetime region V_i. The extrinsic curvature of the shell is given by given by

$$K_{ab}^i = n_{\mu;\nu}^i e_a^\mu e_b^\nu \tag{16}$$

where n_μ denotes the normal to the hypersurface.

Let the energy momentum tensor of the thin shell matter distribution is given by $T^{\mu\nu}$. It can be expressed in the following form

$$T^{\mu\nu} = \delta(\lambda) S^{ab} e_a^\mu e_b^\nu \tag{17}$$

Here S^{ab} is a three dimensional tensor defined over a shell and λ is the Gaussian normal coordinate which takes a specific constant value everywhere on the hypersurface.

The junction conditions for the joining of the two metrics at the thin shell are given as follows. The intrinsic curvature must be continuous across the shell.

$$[h_{ab}] = 0 \tag{18}$$

Here $[A] = A_2 - A_1$. stands for the difference between the quantity A computed on the either side of the hypersurface. The extrinsic curvature is discontinuous across the shell due to the presence of the surface layer of the matter

$$[S_{ab}] = \frac{1}{8\pi}\left([K_{ab}] - [K]h_{ab}\right) \tag{19}$$

K being the trace of the extrinsic curvature.

We now consider a case where the metric defined in the regions V_1, V_2 is Reissner-Nordström and the shell separating these regions is made up of pressureless dust. The shell is electrically neutral and thus the charge parameter

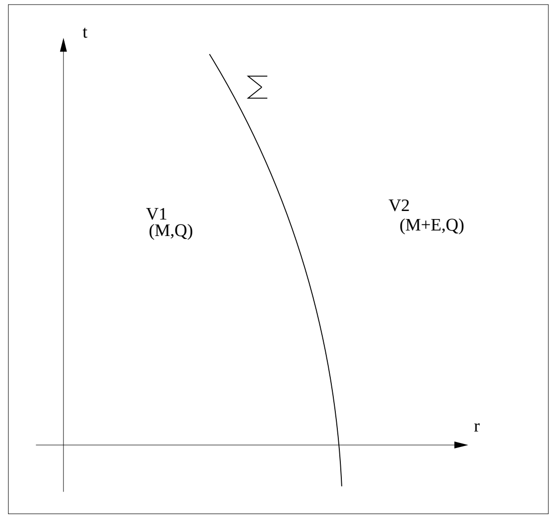

FIG. 2: This is schematic diagram of the spherically symmetric spacetime divided into two parts V_1, V_2 by a thin shell Σ. The spacetime metric in the two parts V_1, V_2 is Reissner-Nordström with different values of mass parameters, namely $M, M+\mu$, but with the same charge Q.

Q associated with both the regions is identical. However the value of the mass parameter would be different in two regions and is given by M and $M+\mu$ with $\mu > 0$. The quantity μ has an interpretation of the energy associated with the shell, since the mass parameter is equivalent to the quasi-local energy.

We use the coordinate systems $x_1^\mu = (t_1, r_1, \theta, \phi)$ and $x_2^\mu = (t_2, r_2, \theta, \phi)$ in the region V_1 and V_2 respectively. The metric in the two regions can be written as

$$\begin{aligned} ds_1^2 &= -f_1(r_1)dt_1^2 + \frac{1}{f_1(r_1)}dr_1^2 + r_1^2 d\Omega^2 \\ ds_2^2 &= -f_2(r_2)dt_2^2 + \frac{2}{f_2(r_2)}dr_2^2 + r_2^2 d\Omega^2 \end{aligned} \tag{20}$$

where

$$f_1(x) = 1 - \frac{2M}{x} + \frac{Q^2}{x^2}, \tag{21}$$

$$f_2(x) = 1 - \frac{2(M+\mu)}{x} + \frac{Q^2}{x^2}. \tag{22}$$

We use the coordinates (τ, θ, ϕ) on the shell. τ is taken to be the proper time for an observer comoving with the shell. Thus the intrinsic metric defined on the shell in this coordinate system would be given by

$$ds_\Sigma^2 = -d\tau^2 + R(\tau)^2 d\Omega^2 \tag{23}$$

Let the equation of the shell defined parametrically from V_1 and V_2 be $t_1 = T_1(\tau), r_2 = R_2(\tau)$ and $t_1 = T_1(\tau), r_2 = R_2(\tau)$. Thus the induced metric computed from the spacetime metric in the ambient spacetime is

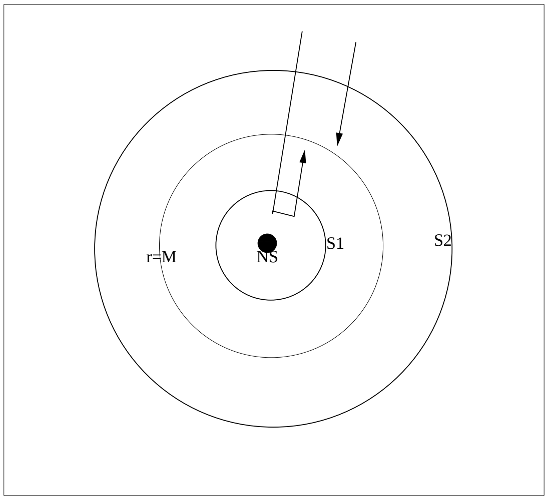

FIG. 3: This is a schematic diagram showing the motion and collision of shells. There is a Reissner-Nordström naked singularity denoted by 'NS' at the center with charge slightly larger than the mass. One of the shell which is initially ingoing s1 turns back as an outgoing shell and then collides with the ingoing shell s2 at $r = M$. The similar picture also can be drawn in case of particle collision replacing shells by particles.

given by

$$ds^2_\Sigma = +\left(-f_1(R_1)\dot{T_1}^2 + \frac{1}{f_1(R_1)}\dot{R_1}^2\right)d\tau^2 + R_1(\tau)^2 d\Omega^2$$
$$ds^2_\Sigma = +\left(-f_2(R_2)\dot{T_2}^2 + \frac{1}{f_2(R_2)}\dot{R_2}^2\right)d\tau^2 + R_1(\tau)^2 d\Omega^2 \quad (24)$$

(23) and (24) taken together imply that

$$R_1(\tau) = R_2(\tau) = R(\tau) \quad (25)$$

and

$$f_1\dot{T}_1 = \sqrt{\dot{R}^2 + f_1} = \beta_1(R, \dot{R})$$
$$f_2\dot{T}_2 = \sqrt{\dot{R}^2 + f_2} = \beta_2(R, \dot{R}) \quad (26)$$

It is clear from (25) that it would be possible to use a single radial coordinate throughout the whole spacetime $r_1 = r_2 = r$. Whereas (26) implies that the time coordinate in the regions V_1, V_2 necessarily have to be different.

The nonvanishing components of the extrinsic curvature as seen from V_1 and V_2 are as follows

$$K^\tau_{1\tau} = \frac{\dot{\beta}_1}{\dot{R}}; K^\theta_{1\theta} = K^\phi_{1\phi} = \frac{\beta_1}{R}$$
$$K^\tau_{2\tau} = \frac{\dot{\beta}_2}{\dot{R}}; K^\theta_{2\theta} = K^\phi_{2\phi} = \frac{\beta_2}{R} \quad (27)$$

The energy-momentum tensor associated with the thin shell is given by

$$T^{\mu\nu} = \sigma\delta(\lambda)u^\mu u^\nu \quad (28)$$

where σ is the surface density and u^μ is the four-velocity of the shell. Comparing it with (17) we get

$$S^{ab} = \sigma u^a u^b \quad (29)$$

Note that the u^a here is the three dimensional tangent vector to the surface as seen by an observer living on the shell unaware of the existence of the four dimensional ambient spacetime in two which the shell is embedded.

We now write down (19) relating the discontinuity in the extrinsic curvature to the matter distribution.

$$-\sigma = \frac{\beta_2 - \beta_1}{4\pi R} \quad (30)$$
$$0 = \frac{\beta_2 - \beta_1}{R} + \frac{\dot{\beta}_2 - \dot{\beta}_1}{\dot{R}} \quad (31)$$

(30) and (31) taken together give

$$m = 4\pi R^2\sigma = constant \quad (32)$$

m is interpreted as the mass of the shell and we also get an equation of motion for the shell which can be written as follows

$$\dot{R}^2 = \frac{1}{m^2}\left(E + \frac{m^2}{2R}\right)^2 - f_1(R) \quad (33)$$

Now we describe the process of shell acceleration and collision. We consider the two concentric spherical thin shells. The spacetime is divided into three regions denoted by V_1, V_2, V_3 by these shell. The metric in the three regions would be given by Reissner-Nordström geometry, however with the different values of parameters in three regions. The shells are assumed to be electrically neutral and thus the charge parameter Q in the three regions would be the same. Mass parameter takes different values in three regions given by $M, M+\mu, M+2\mu$ with $\mu > 0$. The deviation of the mass from the charge of the naked singularity is assumed to be small $0 < \epsilon \ll 1$. We also assume that $M < M + 2\mu < Q$, or equivalently,

$$\mu =: \frac{\epsilon Q}{2}\hat{\mu} \text{ with } 0 < \hat{\mu} < 1 \quad (34)$$

to ensure that the naked singularity does not turn into a black hole.

Hererafter, for simplicity, we assume that the shells are marginally bound, i.e., $\mu = m$. Subscripts s1 and s2 stand for the inner and outer shells, respectively.

Following exactly the same procedure as above the radial component of velocity of the inner as well as outer shells can now be written as

$$\dot{R_{s1}} = \pm\sqrt{\left(1 + \frac{m}{2R_{s1}}\right)^2 - f_1(R_{s1})}$$
$$\dot{R_{s2}} = \pm\sqrt{\left(1 + \frac{m}{2R_{s2}}\right)^2 - f_2(R_{s2})} \quad (35)$$

where f_1 and f_2 are identical to Eqs. (21) and (22), respectively, but with $\mu = m$. $\pm$ stands for outgoing and ingoing shell respectively. Using the normalization condition for velocity $U.U = -1$ we obtain the time components of the velocity as seen from the spacetime region V_2.

$$\dot{T_{s1}} = \sqrt{\frac{1}{f_2(R_{s1})}\left(1 + \frac{\dot{R_{s1}}^2}{f_2(R_{s1})}\right)}$$
$$\dot{T_{s2}} = \sqrt{\frac{1}{f_2(R_{s2})}\left(1 + \frac{\dot{R_{s2}}^2}{f_2(R_{s2})}\right)} \tag{36}$$

The radial component of velocities of both the inner and outer shell would go to zero at infinity. The turning point for the inner shell can be shown to be $R_{s1} = \frac{Q^2 - m^2/4}{2M+m} \approx \frac{Q^2}{2M} \approx \frac{M}{2}$. The turning point for the outer shell can also be shown to be of the same order.

We consider a situation where the inner shell starts off at infinity as an ingoing shell. It then turns back at $R \approx \frac{M}{2}$ and emerges as an outgoing particle. It then collides with the other ingoing shell at $R \approx \frac{Q^2}{M} \approx M$. This situation is exactly analogous to the situation encountered in the previous section.

The center of mass energy of collision between two shells was defined in [9] in a following way generalizing the definition of the center of mass energy of the particles. In case of the particle collisions to compute the center of mass energy, one goes to the orthonormal frame in which the spatial components of the total momentum of the two particles is zero. The time component yields the center of mass energy. While dealing with the collision event of the shells, the center of mass frame was defined to be an orthonormal frame in which the flux of the energy-momentum along the spatial direction is zero and the center of mass energy is defined analogously. It can be shown to be

$$E_{\rm cm}^2 = 2m^2\left(1 - U_{s1}\cdot U_{s2}\right) \tag{37}$$

We compute $U_{s1}\cdot U_{s2}$ in region V_2 for which the expression for the velocities of the shells as seen from V_2, derived earlier and the metric in this region with the parameters $M+\mu$ and Q are used. The center of mass energy of collision at any given value of R turns out to be

$$E_{\rm cm}^2 = 2m^2\left[1 + \frac{1}{f_2}\left(|\dot{R}_{s1}||\dot{R}_{s2}| + \sqrt{(\dot{R}_{s1}^2 + f_2)(\dot{R}_{s2}^2 + f_2)}\right)\right] \tag{38}$$

Here, we assume that a shell is composed of N particles each of which has a mass $\delta m = m/N$. The center of mass energy $E_{\rm p}$ of a collision between two of constituent particles is given by

$$E_{\rm p}^2 = \frac{\delta m^2}{m^2}E_{\rm cm}^2. \tag{39}$$

Using Eq. (34), we have $m = Q\hat{\mu}\epsilon/2$ with $0 < \hat{\mu} < 1$. Then, for $0 < \epsilon \ll 1$, $E_{\rm p}^2$ at $R = Q$ is given by

$$E_{\rm p}^2 \simeq \frac{2\delta m^2}{(2-\hat{\mu})\epsilon}. \tag{40}$$

The above equation seems to imply that the center of mass energy can be indefinitely large. However, in order that the description by a spherical shell is valid, the number of particles N should be much larger than unity, i. e.,

$$N = \frac{m}{\delta m} = \frac{Q\hat{\mu}}{2\delta m}\epsilon \gg 1, \tag{41}$$

or equivalently,

$$\epsilon \gg \frac{2\delta m}{Q\hat{\mu}}. \tag{42}$$

Due to this constraint, we have

$$\begin{aligned} E_{\rm p} &\ll \sqrt{\frac{\hat{\mu}}{2-\hat{\mu}}\delta m M} < \sqrt{\delta m M} \\ &= 3.24\times 10^{28}\left(\frac{\delta m}{m_{\rm p}}\right)^{\frac{1}{2}}\left(\frac{M}{M_\odot}\right)^{\frac{1}{2}}\ {\rm GeV}, \end{aligned} \tag{43}$$

where we have used $Q \simeq M$. The above equation implies that if M is order of the solar mass $M_\odot = 1.99\times 10^{30}$kg, the center of mass energy $E_{\rm p}$ can exceed Planck scale $m_{\rm pl} = \sqrt{hc/2G} = 2.16\times 10^{19}$GeV even if δm is the order of the proton mass $m_{\rm p} = 0.938$GeV.

IV. CONCLUSIONS

In this paper we studied the particle and shell acceleration by Reissner-Nordström naked singularities. The phenomenon of particle acceleration and collision with extremely large center of mass energy was previously studied and explored in the background of extremal and near extremal black holes. We extended this result to the near extremal naked singularities. We showed that there are significant qualitative differences in the particle acceleration mechanism by black holes and naked

singularities. In case of black holes, the particle collision is between ingoing particles, and to achieve large center of mass energy of collision, fine tuning of parameters is necessary, and proper time required from such a collision to take place is infinite in the rest frame of one of the colliding particles. On the contrary, in case of naked singularity, it is possible to consider a collision between ingoing and outgoing particles, since due to the absence of the event horizon and the repulsive gravity effects near singularity, initially ingoing particle turns back as an outgoing particle. This eliminates the necessity of the fine tuning of the parameters and also the proper time required for the collision to take place also happens to be finite. Particles participating in the collision are assumed to be test particles following the geodesics on the background geometry. The effects of gravity generated by the particles are ignored.

Thus to study whether or not the phenomenon of divergence of center of mass energy survives, we studied the collision between the concentric spherical concentric shells. The gravity of the shells is taken into account in an exact calculation, and the center of mass energy of collision of shells is computed in a situation analogous to the test particle case. It is shown that, in this case, due to the condition that the outermost region is described by the over-charged RN spacetime, the center of mass energy of a collision between two of the constituent particles of the shells is bounded above. However, if the mass of the central naked singularity is order of the solar mass and if the mass of a constituent particle of the shells is order of the proton mass, the upper bound is order of 10^{28}GeV, and hence the center of mass energy can easily exceed the Planck scale.

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